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Conditioning analysis of nonlocal integral operators in fractional Sobolev spaces

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Abstract

We study the conditioning of nonlocal integral operators with singular and integrable kernels in fractional Sobolev spaces. These operators are used, for instance, in peridynamics formulation and nonlocal diffusion. In 1D, we present sharp quantification of the extremal eigenvalues in all three parameters: size of nonlocality, mesh size, and regularity of the fractional Sobolev space. We accomplish sharpness both rigorously and numerically.

For the minimal eigenvalue, we obtain sharpness analytically by using a nonlocal characterization of Sobolev spaces. We verify this estimate by exploiting the Toeplitz property of the stiffness matrix. However, the analytical approach fails to give sharp quantification of the maximal eigenvalue. Hence, in 1D, we take an algebraic approach by directly working with the stiffness matrix entries, which have complicated expressions due to all three parameters. We systematically characterize the nonzero entries and dramatically simplify their expressions by using convenient algebra. We establish the zero row sum property of the stiffness matrix and negativity of the off-diagonal entries. Eventually, we arrive at sharpness through the use of the Gerschgorin circle theorem.